Superconductivity - M1

Jérôme Lesueur - Kris Van Houcke

TD 3: Fundamental properties of type I superconductors

1 Thermodynamics of type I superconductors

Consider a long cylinder in a magnetic field parallel to its axis. Since we are working at constant external field H [1], the relevant thermodynamic potential is the Gibbs free energy (in french: 'enthalpie libre'):

$$G(T,H) = F(T,B) - \int d\mathbf{r}\ BH\ , \tag{1}$$

where F is the Helmholtz Free energy (in french: 'énergie libre'). At the superconducting transition, the Gibbs free energies per unit of volume of the normal and superconducting phase must be equal:

$$g_n(T,H_c) = g_s(T,H_c)\ . \tag{2}$$

This equality determines the thermodynamical critical field $H_c(T)$ of the transition. (Note that there is no phase transition at H_c in a so-called type II superconductor. Discussion of type II will follow in a later TD).

1. Give the free energy difference $f_n(T,B=0) - f_s(T,B=0)$ as a function of $H_c(T)$. Assume that the radius of the cylinder is much larger than the penetration depth.

2. The quantity of the previous question is the *condensation energy*. It is a measure of the gain in free energy per unit volume in the superconducting state compared with the normal state at the same temperature. Let us take Niobium as an example. Here $T_c = 9$ K and $H_c = 160$ kA m^{-1} ($B_c = \mu_0 H_c = 0.2$ T). Nb has a bcc crystal structure with a 0.33 nm lattice constant. Calculate the condensation energy per atom. Is this a big number?

3. Derive an expression for the entropy difference of the normal state and the superconducting state as a function of the critical field. In presence of a magnetic field, the transition is of first order. Give an expression for the latent heat. Does the system absorb or emit heat when going from the superconducting to the normal state?

4. Give an expression for the specific heat difference $C_n - C_s$. What is the jump in the specific heat at the transition in absence of a magnetic field (when the transition is of second order)?

[1] To get a physical feeling for $\mathbf{H}$, consider the situation when there are no external currents fed into the sample. One can then write for the current density: $\mathbf{J}(\mathbf{r}) = \mathbf{J}_s(\mathbf{r}) + \mathbf{J}_{ext}(\mathbf{r})$, with $\mathbf{J}_s(\mathbf{r})$ the supercurrent density at point $\mathbf{r}$ in the sample ($\mathbf{J}_s = 0$ outside) and $\mathbf{J}_{ext}$ is the current density in the coils that create the field ($\mathbf{J}_{ext} = 0$ inside the sample). All currents contribute to $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$, but only $\mathbf{J}_{ext}$ contributes to $\nabla \times \mathbf{H} = \mathbf{J}_{ext}$. One might be tempted to say or think that $\mathbf{H}$ is the field that would exist if the sample would not be there (for the same $\mathbf{J}_{ext}$ in the coils). This is correct for certain geometries (like the cylindrical one), but *it is wrong in general*! For a spherical sample, for example, the field lines are distorted around the sphere, and this implies a change of $\mathbf{H}(\mathbf{r})$ (while $\nabla \times \mathbf{H} = \mathbf{J}_{ext}$ remains).

The specific heat per unit of volume of a metal in the superconducting and normal state is given by

$$C = aT^3 \,, (T < T_c) \tag{3}$$
$$C = bT^3 + \gamma T \,, (T > T_c) \,, \tag{4}$$

where a and b are constants that can be related to the spectrum of phonons of the metal and γ is the Sommerfeld constant. Use this parametrisation and show that

4. the transition temperature in absence of external magnetic field is given by

$$T_c = \sqrt{\frac{3\gamma}{a-b}} \,. \tag{5}$$

5. the temperature dependence of the critical magnetic field is

$$H_c(T) = H_c(0)\left(1 - (T/T_c)^2\right) \,, \tag{6}$$

with $H_c(0) = T_c\sqrt{\gamma/(2\mu_0)}$.

6. the difference of the internal energy of the two states in absence of external field reaches a maximum for $T = T_c/\sqrt{3}$.

2 Critical current of a superconducting wire

1. Consider an infinitely long superconducting wire of cylindrical shape with radius R with $R \gg \lambda_L$. The Silsbee criterion states that the critical value of the current to destroy superconductivity is that at which the magnetic field due to the current itself is equal to the critical magnetic field. Give the critical current I_c as a function of the radius R and the critical magnetic field H_c .

2. Does the critical current scale with the cross-sectional area of the wire? What does this imply about where the current flows in the wire? What would the London theory predict for the critical current density in this case (answer without trying to solve the London equations explicitly for this geometry)?

3. To achieve a high critical current, is it better to use one thick wire or to use many thin wires with the same total cross-section? What about conventional conducting wires?

4. Compute the value of the critical current I_c for a lead wire of radius $R = 1$ mm at $T = 4.2$ K. Lead has a critical temperature of $T_c = 7.2K$ and a critical field of $H_c(T=0) = 800\ Oe$. The temperature dependence of the critical field is $H_c(T) = H_c(0)\left(1 - (T/T_c)^2\right)$. Comment on the value of I_c .

3 Intermediate state above the critical current in a superconducting wire

We consider here a cylindrical wire of radius $R \gg \lambda$ carrying a current $I > I_c$ exceeding the critical value.

1. Based on Silsbee's rule, if $I > I_c$, then the surface field exceeds H_c , and at least the surface must become normal. If a surface layer were to go normal then we are left with a fully superconducting core through which all the current flows. Is such a configuration stable?

2. On the other hand, what if the sample went entirely normal? Is this a stable configuration? (Hint: calculate the field $H(r)$ as a function of the radius.)

These observations suggest a so-called *intermediate state*, where both normal and superconducting regions coexist. From the above questions, we conclude that the nature of the intermediate (mixed) state in a region $r < R_0$ is dictated by the requirement that $H(r) = H_c$. For $R_0 < r < R$ the wire is in the normal state. The normal regions, with resistivity ρ , have to carry some current, otherwise we end up again with an unstable configuration. These requirements are approximately reconciled by the configuration shown in Fig. 1, first proposed by F. London, in which the fractional path length (parallel to the axis of the wire) of resistive material is r/R_0 .

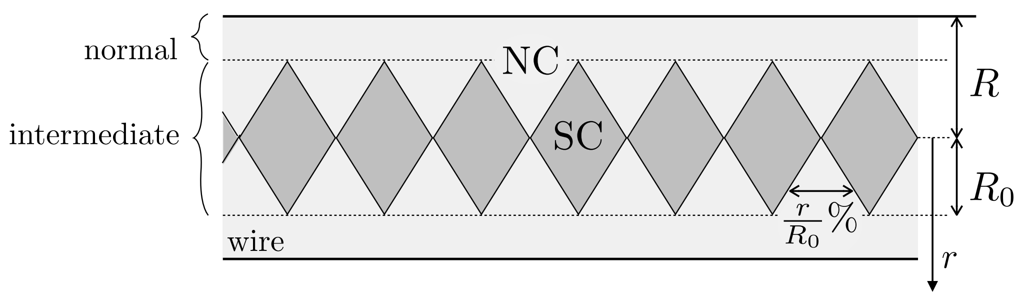

Figure 1: London's model of the intermediate-state structure in a wire carrying a current in excess of I_c . The dark grey region is superconducting (SC) while the light grey region is normal (NC).

Any geometry, except an infinitely long cylinder in a field parallel to its axis, has such an intermediate state. In many shapes (due to the demagnetisation factor) there is already an intermediate state for an external field $H < H_c$.