Problem 1 Union of groups

Let G_1 and G_2 be two subgroups of G. In the lecture you saw that the intersection of two subgroups always defines a group. When does the union $G_1 \cup G_2$ give rise to a group?

Problem 2 Cayley tables of finite groups

1. Construct the Cayley tables of all finite groups up to order 5. Show that they are Abelian. Classify the different groups by isomorphisms to (products of) the cyclic groups $\mathbb{Z}/n\mathbb{Z}$.
2. Build the Cayley table of the group D_3. Is the group Abelian? Identify all subgroups and verify Cayley's theorem. Whenever possible, construct the corresponding quotient group and its Cayley table. Find the left- and right cosets of some non-normal subgroup.

Problem 3 Quotient groups

1. Show that if $A \leq G/H$, then there exists a $G' \leq G$ such that $H \triangleleft G' \leq G$ and $A = G'/H$.
2. Show that

$$G/Z(G) \simeq \mathrm{Inn(G)},$$

where Inn(G) is the group of inner automorphisms of G. An inner automorphism is defined as

$$\begin{aligned} \varphi_g : G &\to G \\ h &\mapsto \varphi_g(h) = g \cdot h \cdot g^{-1}. \end{aligned}$$