Nonlinear Physics

Systems governed by nonlinear equations display multiple solutions with different symmetries. We study the bifurcations i.e. the transitions between these solutions when a parameter of the system is varied. We show that in the vicinity of these bifurcations, the system is governed by universal equations, normal forms, that mostly depend on the broken symmetries at the transition. We emphasize the analogy with phase transitions, but also point out differences such that limit cycles or chaotic behaviors that do not occur at equilibrium.

1. An introduction to nonlinear phenomena.
Simple examples using electric or mechanic devices.

2. Nonlinear oscillators: quasilinear versus relaxation regimes. Frequency locking. Parametric resonance and related topics.

3. Adiabatic elimination of damped modes. Normal forms for elementary bifurcations.

4. Broken symmetries and amplitude equations.
Pattern forming instabilities in hydrodynamics. Analogy with the mean field description of second order phase transitions.

5. Broken symmetries and neutral modes. Phase dynamics.

6. Subcritical bifurcations and metastable states. Localized structures. Analogy with the liquid-vapor transition. Nucleation and Maxwell construction. Non potential effects.