Lie Groups, Lie Algebras And Representations

Master 2 ICFP 2nd

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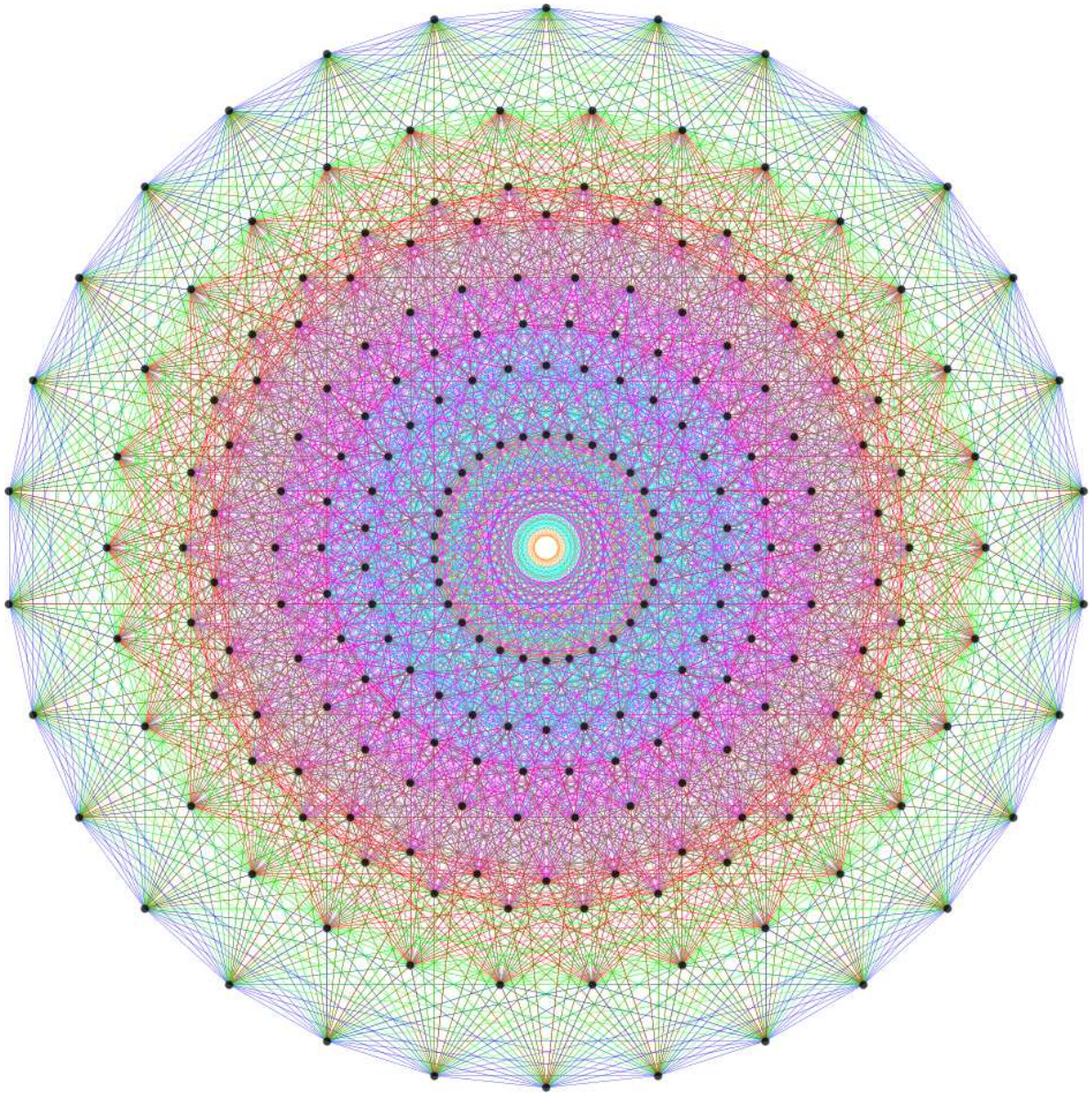
Lie Groups, Lie Algebras and Representations

- **Theory of groups and their representations** : central subject about symmetries in mathematics and in physics.
- Science of **symmetries**.
- Crucial examples :
- **Finite groups** : symmetric groups S_n, isometry groups...
- **Lie groups** : $SL_2(\mathbb{C}) \subset GL_2(\mathbb{C})$, $U_n(\mathbb{C})$...

Lie Groups, Lie Algebras and Representations

- **Representation** : action of a group G on a vector space V by linear operators
- Group morphism : $G \to GL(V)$
- Various interesting properties depending on G, V, the based field...
- **Many applications** in mathematics and physics.
- Quantum Physics : state space as a representation of observable algebra, classification of particles...
- Central theme in mathematics since 19th century

Lie Groups, Lie Algebras and Representations



Lie Groups, Lie Algebras and Representations

- Aim of this course : introduction to classical concepts and tools in Lie theory as well as for the theory of their representations.
- We will study remarkable examples (in particular for the applications in physics).

- Plan of the course

- **1. Representations of groups and algebras**.
  Generalities. Finite groups and their characters.
- **2. Groups and Lie algebras of finite dimension**.
  Lie groups, reminders on differential geometry. Lie algebras. Fundamental examples, Heisenberg algebras. Semi-simple Lie algebras. Categories of representations, irreducible representations. Complete reducibility. Structure of semi-simple Lie algebras. Root systems, Weyl group.

- **3. Representations of finite-dimensional Lie algebras**. Highest weight modules, Verma modules, category O. Parametrization of simple modules. Jordan-Holder series, multiplicities. Finite-dimensional representations. Tensor structure, characters.
- **4. Generalizations and discussions**. Compact Lie groups and their representations. Spin representations. Loop algebras, central extensions, integrable representations. Virasoro algebras.

- There is no special prerequisites, except standard linear and standard general algebra.
- Course : Thursday 9 - 12
- Exercise classes : Monday 2 - 4
- **See you Thursday !**