Echoes from the Cosmos

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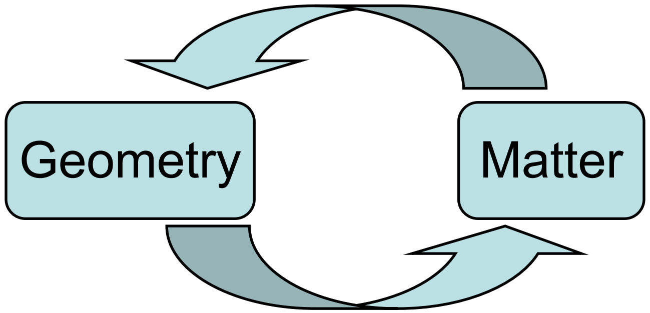
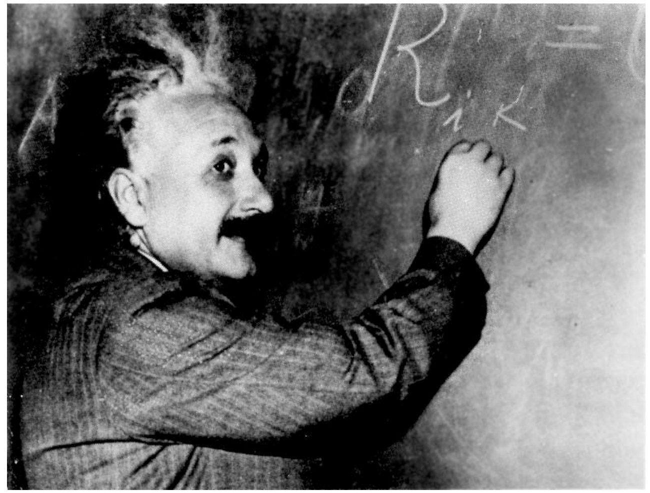
Astroparticules
et Cosmologie

Modern cosmology: basics

- **General relativity (1915)**



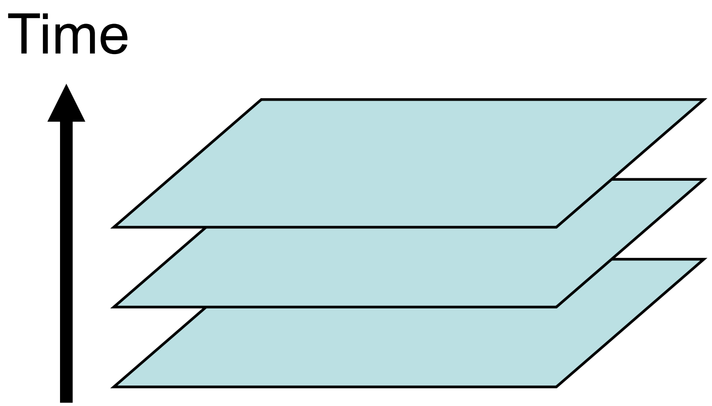


$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

- **Cosmology**

  homogeneous and isotropic

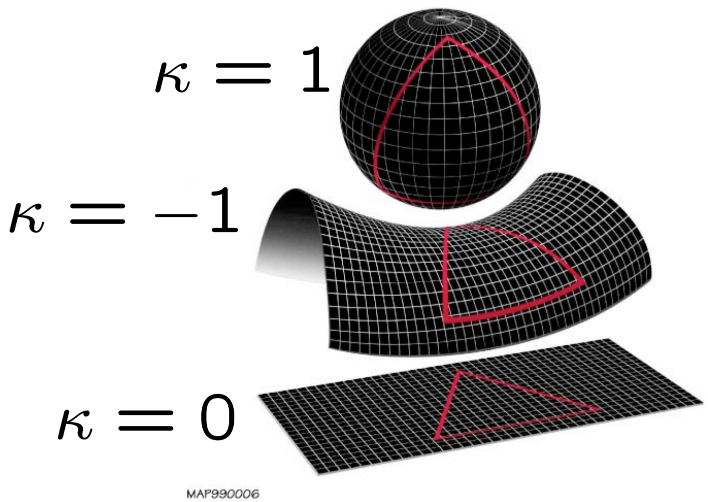
  spatial slices



Geometry of spacetime

- **Spatial dependence**

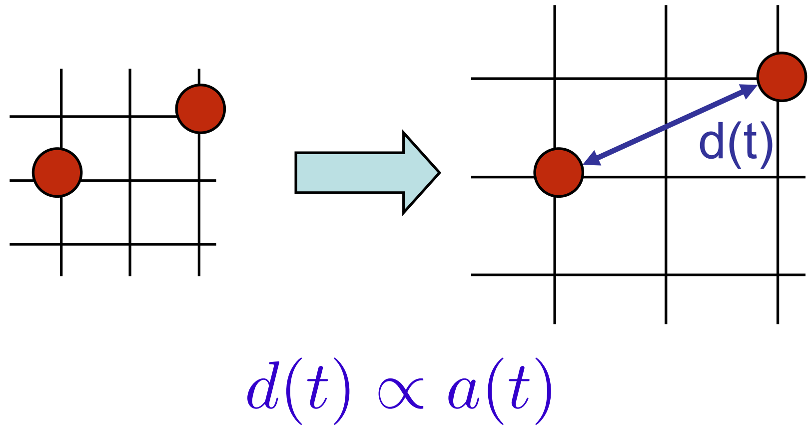
3 possibilities:



- **Time dependence**

a(t): scale factor

Between two points **at rest**



Standard cosmological model

- **FLRW geometry** (spatial homogeneity and isotropy)

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right]$$

- **Matter content**

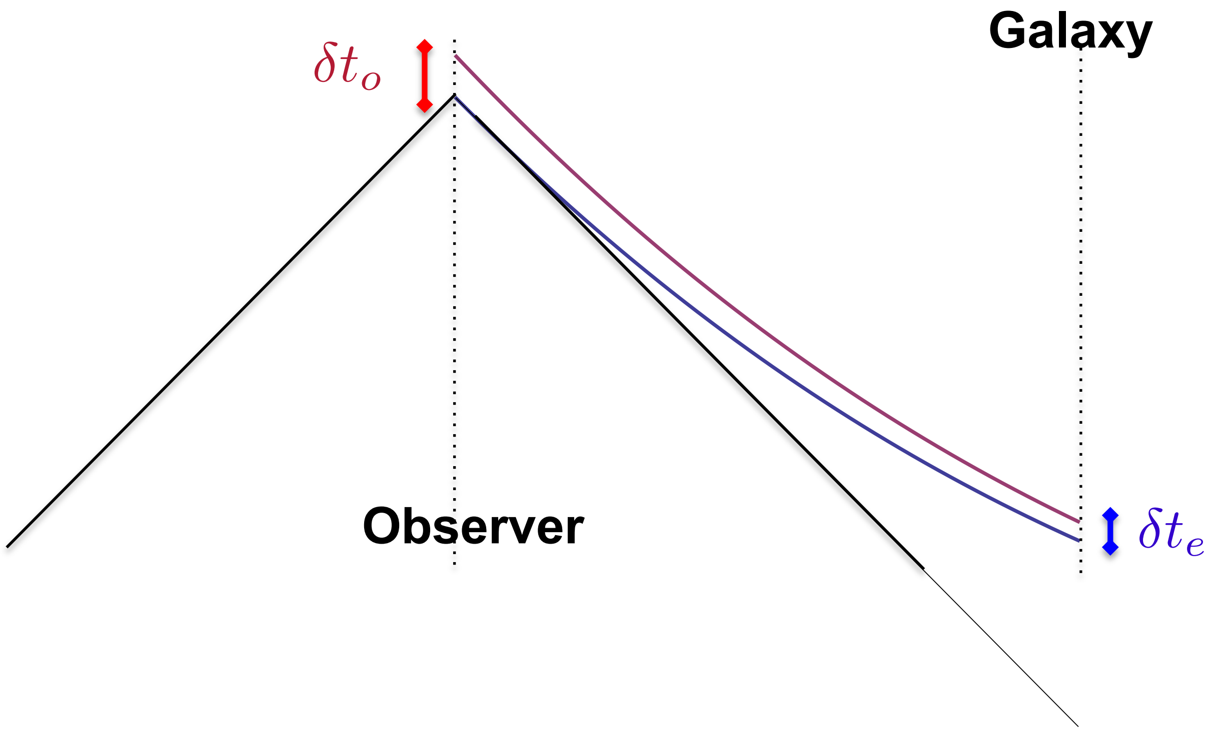
$$T^\mu_\nu = \mathrm{Diag}(-\rho, P, P, P)$$

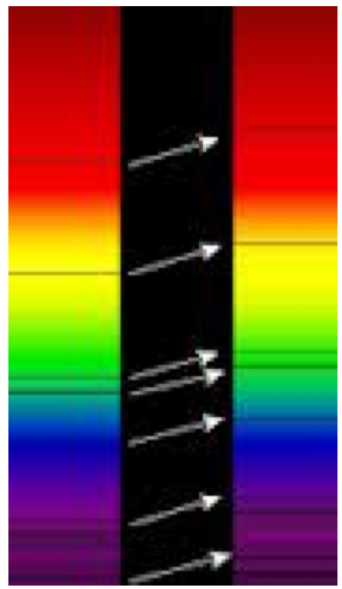
- **Friedmann equations**

$$R_{\mu\nu} - \frac{1}{2}R\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu} \quad \Rightarrow \quad \begin{aligned} \left(\frac{\dot a}{a}\right)^2 &= \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2} \\ \frac{\ddot a}{a} &= -\frac{4\pi G}{3}(\rho + 3P) \end{aligned}$$

Expansion history

- Our past light-cone is "deformed": $dt = \pm\, a(t) dr$





- One can measure the **redshift**: $$\frac{\delta t_o}{\delta t_e} = \frac{\lambda_o}{\lambda_e} = \frac{1}{a_e} = 1 + z$$

Expansion history

- One can determine $a(t)$ by measuring the redshifts and distances of many sources.

- To measure distances, two types of ideal objects:
  - "Standard candles"

$$\frac{dE_{\rm obs}}{dt_{\rm obs}\, dS_{\rm obs}} \leftarrow F_{\rm obs} = \frac{L_{\rm int}}{4\pi D_L^2}, \quad L_{\rm int} \rightarrow \frac{dE_e}{dt_e}$$

$$D_L(z) = (1+z)\, r(z)$$

  - "Standard rulers"

$$\Delta\theta = \frac{d}{D_A}$$

$$D_A(z) = \frac{r(z)}{1+z}$$

- In practice,
  - **Supernovae Ia:** ``standardizable'' candles
  - BAO scale used as a standard ruler

Accelerated expansion

- Assuming only ordinary matter $\dot{a}^2 \propto a^2 \rho_m \propto 1/a$
  one gets a **decelerated expansion.**

- **Cosmological constant** (Einstein, 1917)

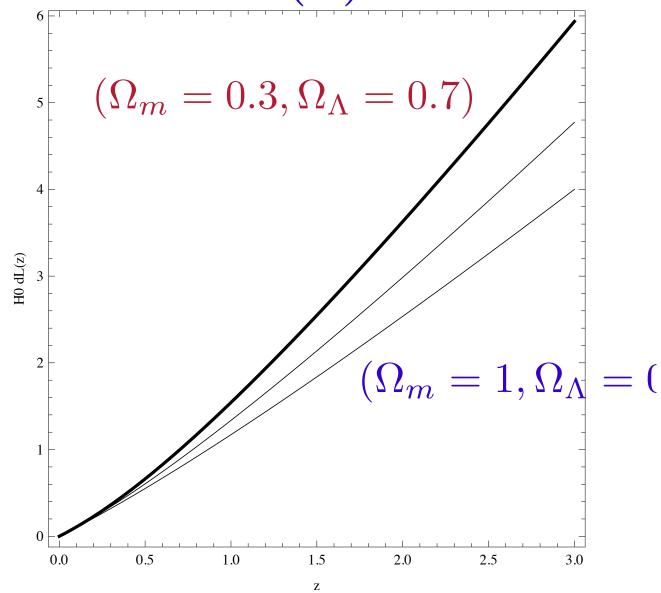
$$G_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

Modified Friedmann equation

$$3\frac{\dot{a}^2}{a^2} = 8\pi G\rho + \Lambda \implies \dot{a}^2 \sim \Lambda a^2$$

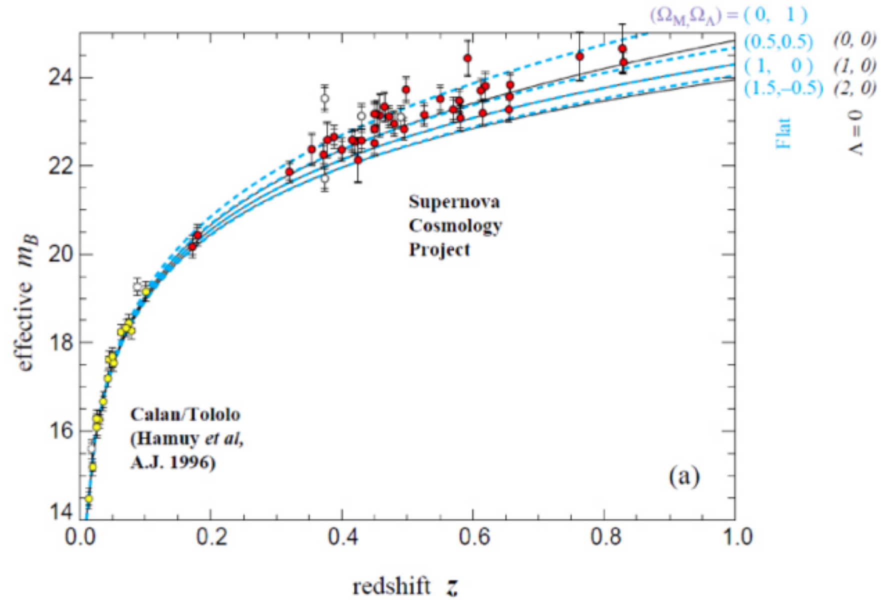
Acceleration expansion !

$D_L(z)$



Accelerated expansion

Analysis of Supernovae (1998)



$$\Omega_m = \frac{\rho_{\rm m}}{\rho_{\rm tot}} \simeq 0.3$$

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{\rm tot}} \simeq 0.7$$

Our universe is accelerating !

Nobel Prize 2011

Saul Perlmutter

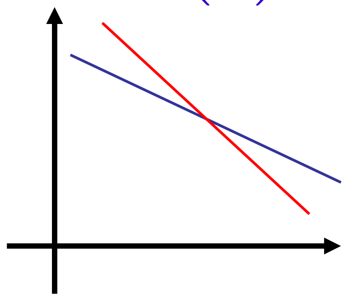
Brian P. Schmidt

Adam G. Riess

The Universe in the Past

The energy densities dilute at various rates

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$



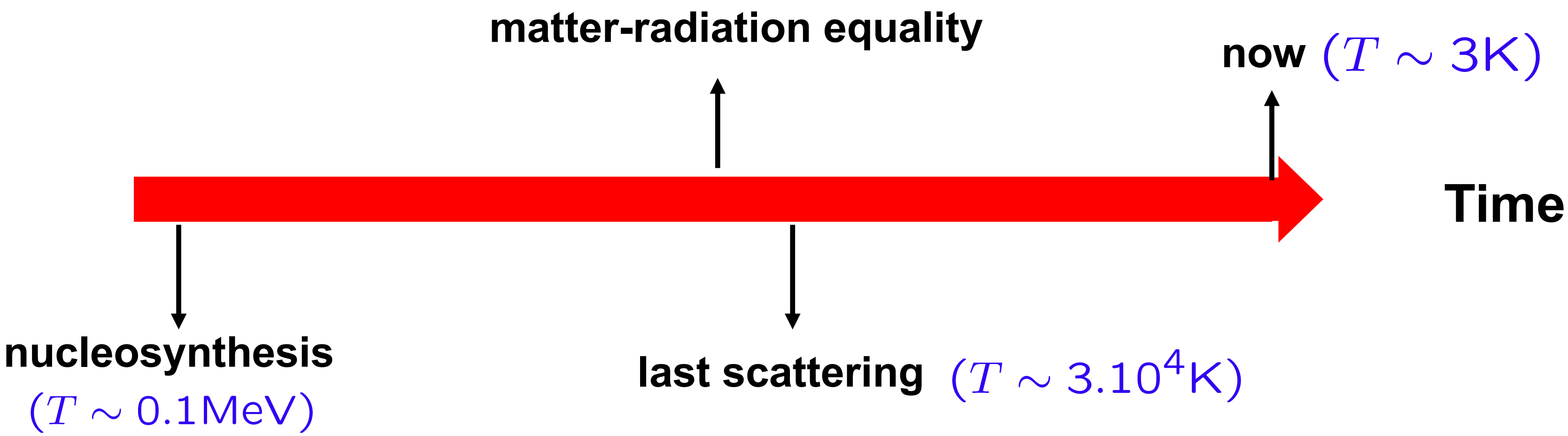
- pressureless matter

$$\rho_m \propto \frac{1}{a^3} \qquad \Rightarrow a(t) \propto t^{2/3}$$

- radiation

$$\rho_r \propto \frac{1}{a^4} \qquad \Rightarrow a(t) \propto t^{1/2}$$

$$T \propto 1/a$$



Cosmic Microwave Background (CMB)

- $T > 3.10^3$ K: atoms are ionised ($H \rightarrow p+e^-$)
  opaque Universe

- $T < 3.10^3$ K: "recombination" ($p+e \rightarrow H$)
  transparent Universe

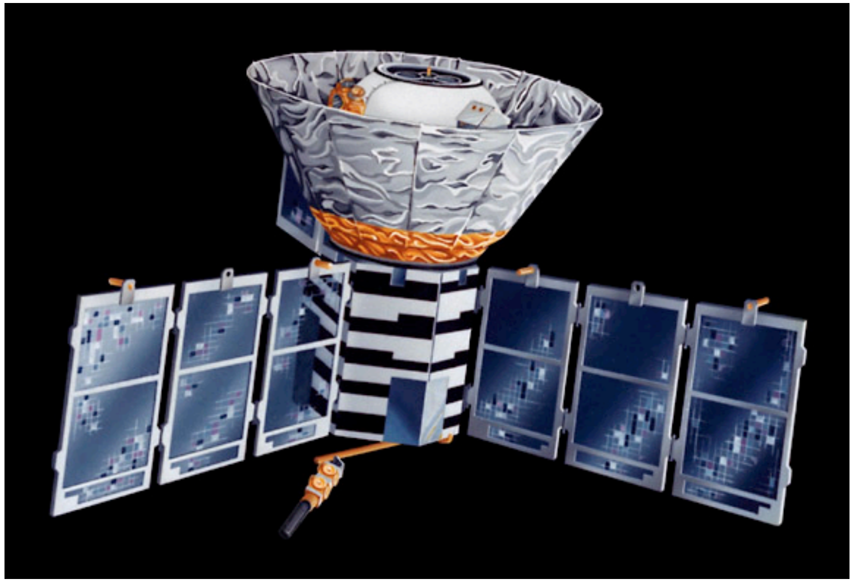
- **"Fossil" background radiation**
  – predicted in the 1940's,
  – discovered in 1964 by **Penzias and Wilson**.

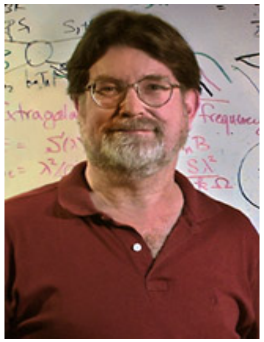
*« A possible explanation for the observed excess noise temperature is the one given by Dicke, **Peebles**, Roll, and Wilkinson (1965) in a companion letter in this issue. »*

$T_0 \approx 2.7$ K

Nobel Prize 1978

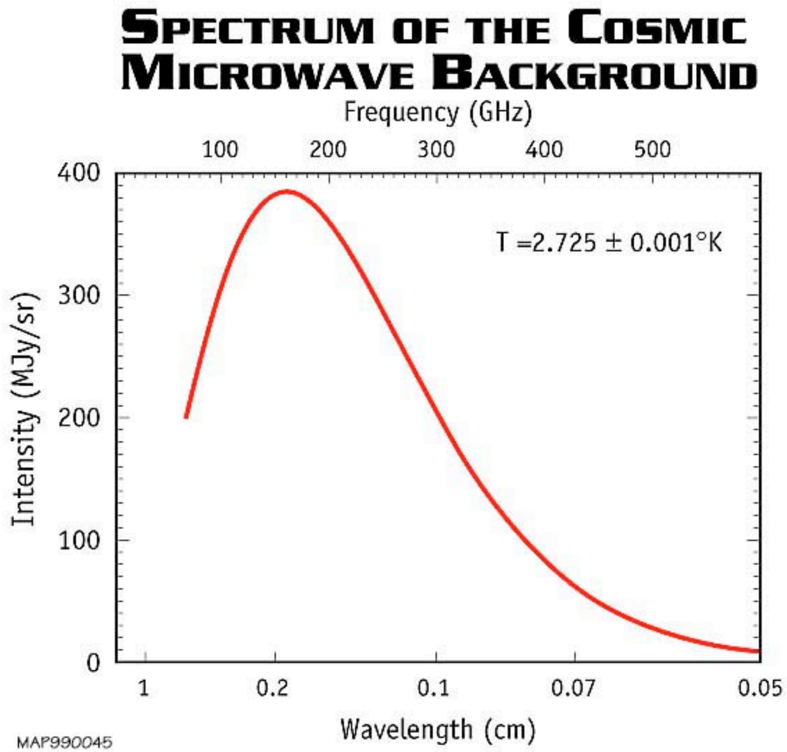
CMB seen by COBE





John C. Mather

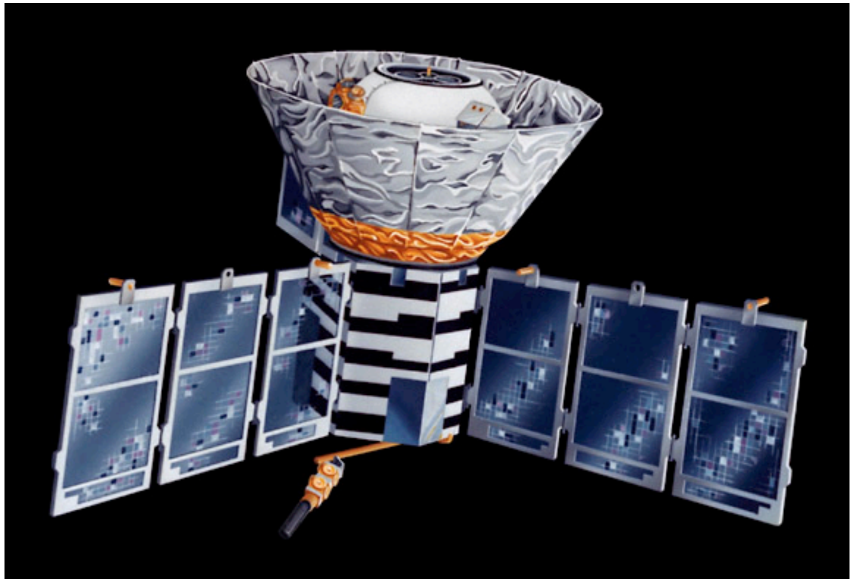
George F. Smoot



"for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation"

Nobel Prize 2006

CMB seen by COBE (1992)



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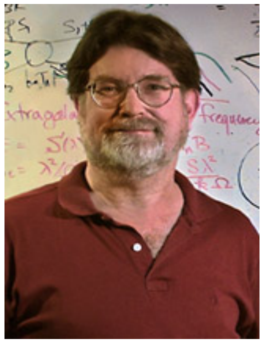
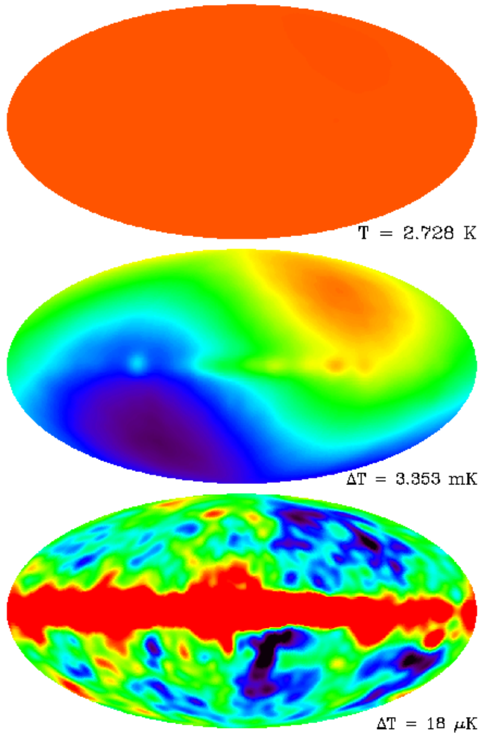


George F. Smoot

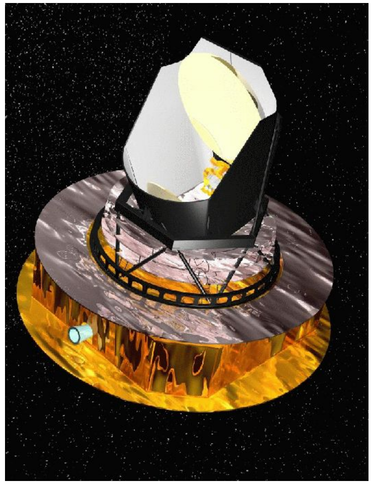
Nobel Prize 2006

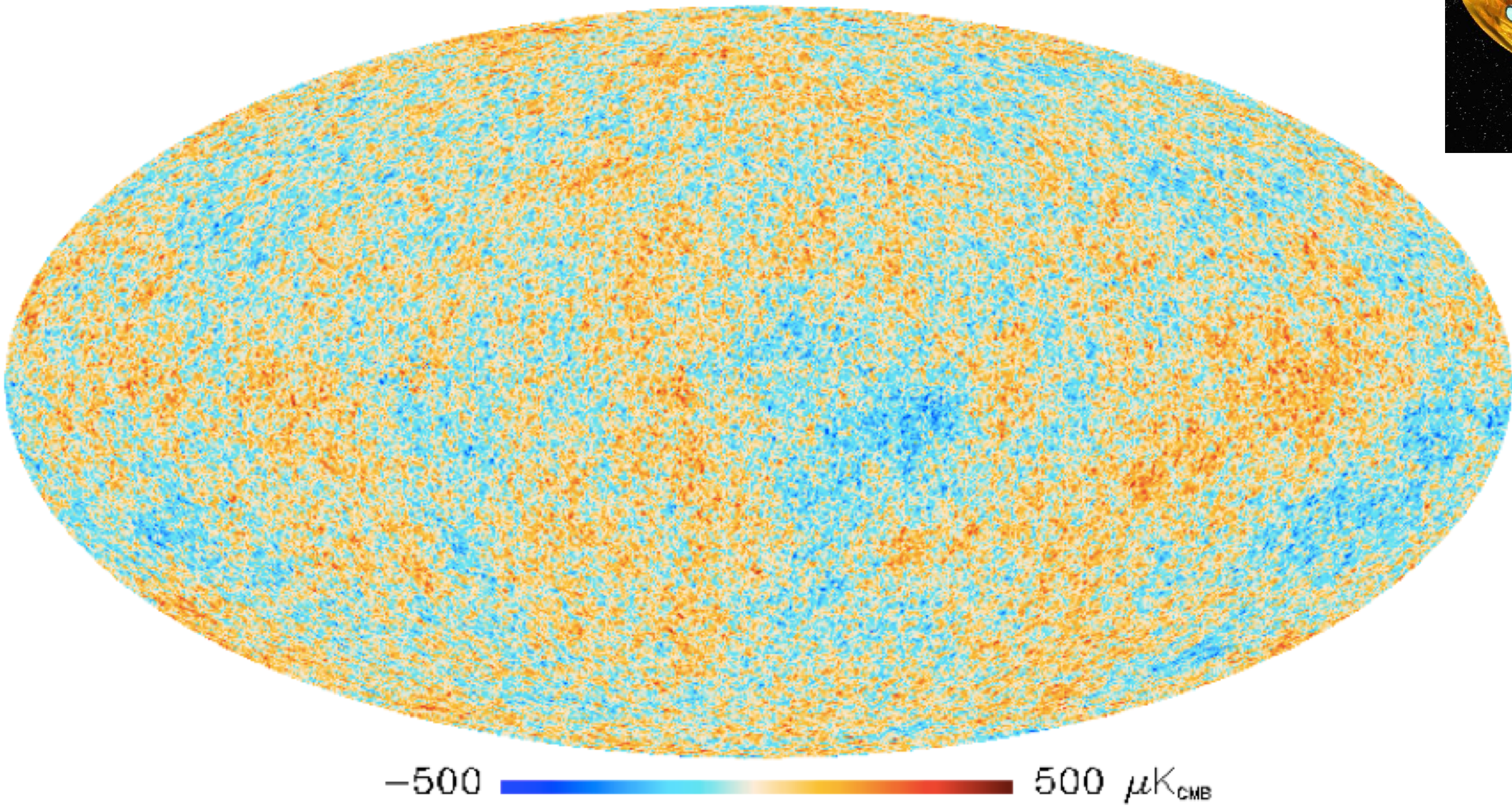


$$\frac{\delta T}{T} \sim 10^{-5}$$

CMB seen by Planck (2013)

$$\frac{\delta T}{T} \sim 10^{-5}$$





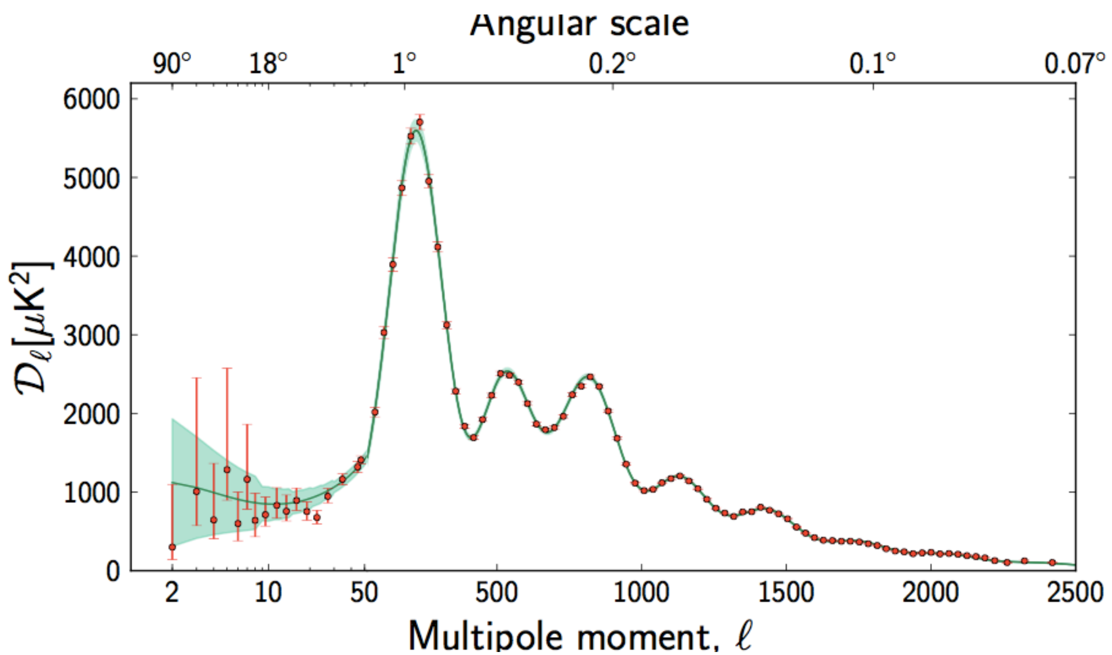
CMB seen by … theorists

- Decomposition in spherical harmonics

$$\frac{\delta T}{T}(\theta, \phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi)$$

$$C_l = \langle |a_{lm}|^2 \rangle$$

$$\mathcal{D}_\ell = \frac{\ell(\ell+1)}{2\pi} C_\ell$$



Oscillations = "frozen" sound waves

Primordial fluctuations + cosmological parameters ➡ CMB predictions

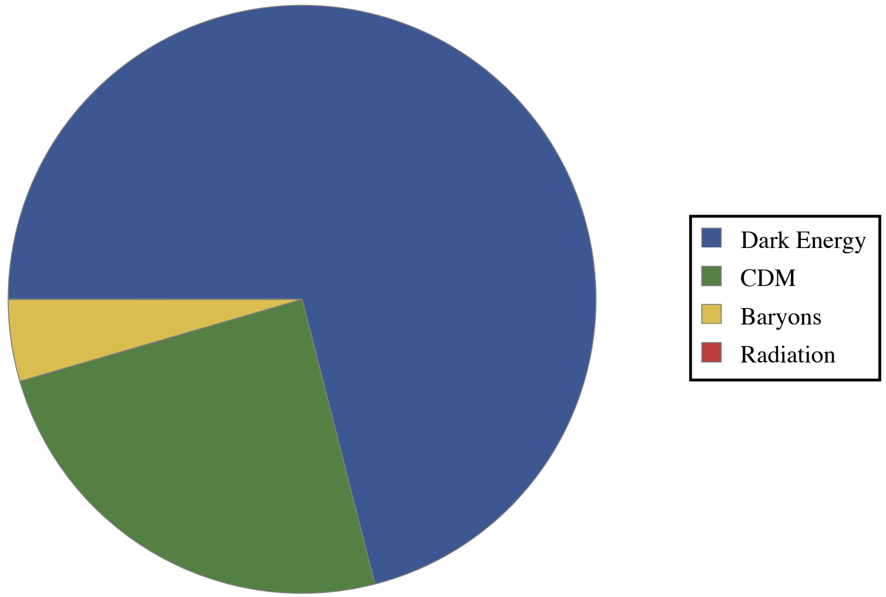
Cosmological parameters

- **Standard model** (confirmed by observations)
  - Spatial geometry is Euclidean $\kappa = 0$
  - Five components: baryons, photons, neutrinos, **dark matter + cosmological constant**

$$\Omega_b + \Omega_c + \Omega_\Lambda + \Omega_\gamma + \Omega_\nu = 1 \qquad \Omega_s \equiv \frac{8\pi G}{3H_0^2}\rho_s$$

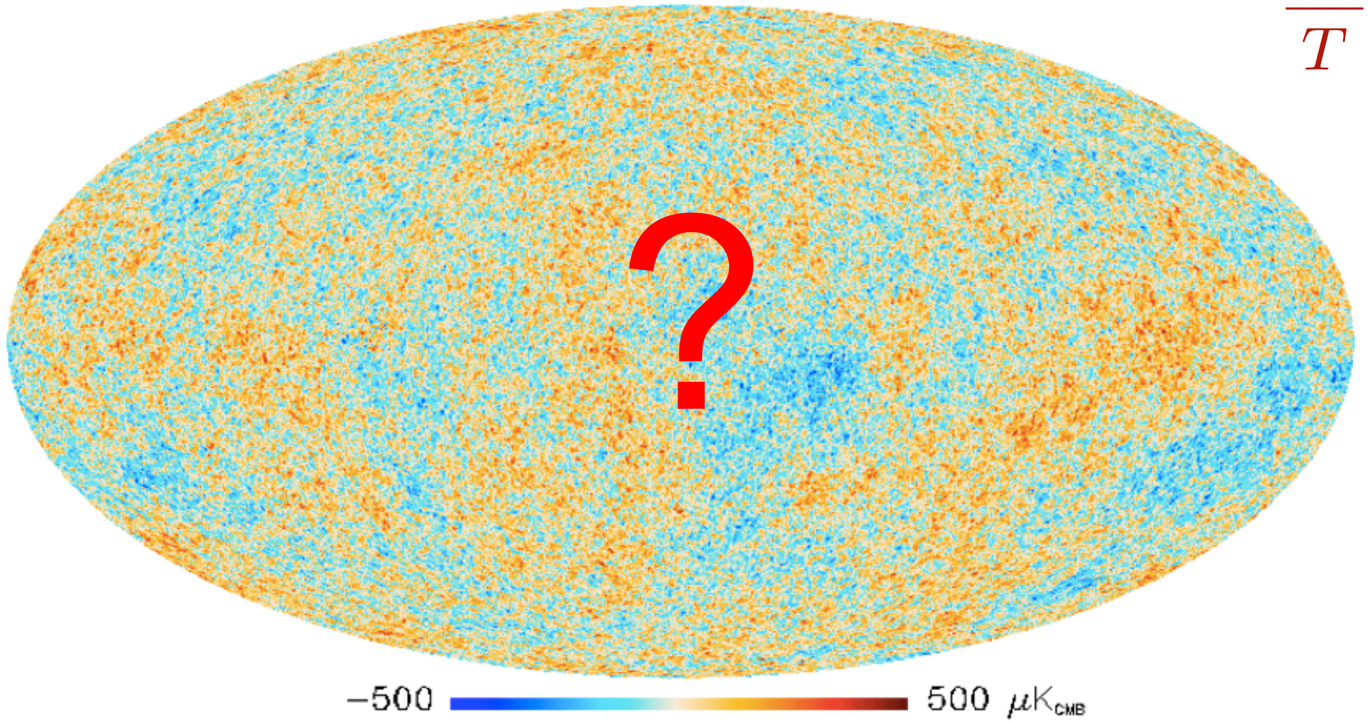
$$H_0 \equiv \left(\frac{\dot{a}}{a}\right)(t_0)$$

- Present "recipe" for our Universe



Origin of the primordial fluctuations ?

$$\frac{\delta T}{T} \sim 10^{-5}$$



Main scenario:

- Phase of **inflation** in the early universe
- Amplification of the quantum fluctuations of a scalar field

Cosmological perturbations

- Linear perturbations: $f(t,\vec{x}) = \bar{f}(t) + \delta f(t,\vec{x})$

described by a homogeneous and isotropic Gaussian random field

$$\langle f(\vec{x}_1) f(\vec{x}_2) \rangle = C_f(\|\vec{x}_1 - \vec{x}_2\|)$$

- At linear level, Fourier modes are independent

$$\tilde{f}(\vec{k}) = \int \frac{d^3x}{(2\pi)^{3/2}}\, e^{-i\vec{k}\cdot\vec{x}} f(\vec{x})$$

- **Power spectrum**

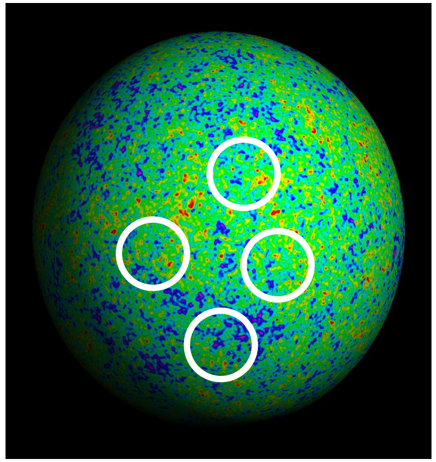
$$\langle \tilde{f}(\vec{k}_1) \tilde{f}^*(\vec{k}_2) \rangle = 2\pi^2 k^{-3} \mathcal{P}_f(k_1) \delta(\vec{k}_1 - \vec{k}_2)$$

$$\langle f(\vec{x}_1) f(\vec{x}_2) \rangle = \int \frac{d^3k}{4\pi k^3}\, e^{i\vec{k}\cdot(\vec{x}_1 - \vec{x}_2)} \mathcal{P}_f(k)$$

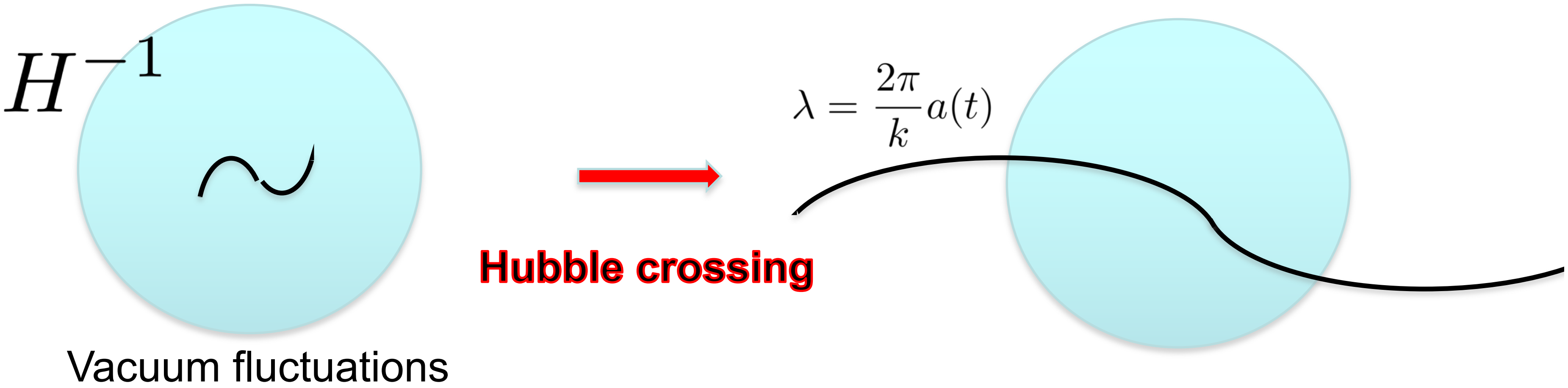
Inflation

- A period of acceleration in the early Universe

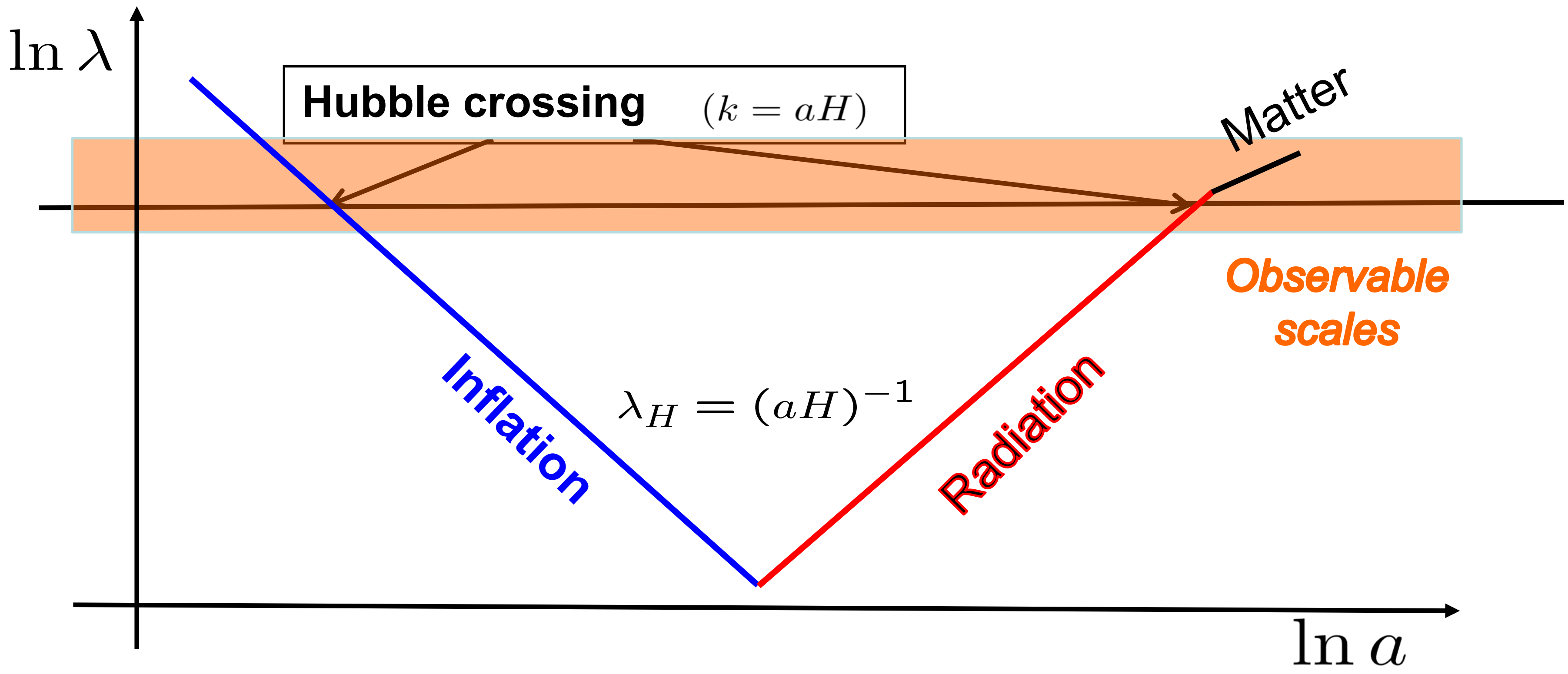
$$\ddot{a} > 0$$



- Solves the horizon and flatness problems
- Origin of the primordial perturbations



Sub- and super-Hubble scales



- **Hubble crossing** for a (comoving) scale λ

$$\lambda \sim \lambda_H \quad \Longleftrightarrow \quad k = aH$$

- Larger scales cross out the Hubble radius **earlier** during inflation and cross inside the Hubble radius **later** in the standard era.

Scalar field inflation

- How to get inflation ? $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)\,.$$

- Scalar field $$S_\phi = \int d^4x\sqrt{-g}\left(-\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)\right)$$

- Homogeneous equations

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

$$3H^2 = 8\pi G\,\rho \qquad \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

- Slow-roll motion $$\frac{1}{2}\dot{\phi}^2 \ll V(\phi), \quad \ddot{\phi} \ll 3H\dot{\phi}$$

$$P \simeq -\rho$$

Perturbations during inflation

- Scalar field fluctuations: $$\phi(t,\vec{x}) = \bar{\phi}(t) + \delta\phi(t,\vec{x})$$
- The quantum fluctuations of the scalar field are amplified at **Hubble crossing** $(k = aH)$ $$\delta\phi \simeq \frac{H}{2\pi}$$
- This generates geometrical perturbations $$\Psi \sim \frac{\delta a}{a} \sim \frac{\dot{a}}{a}\delta t \sim H\delta t \sim H\frac{\delta\phi}{\dot{\phi}} \sim \frac{H^2}{\dot{\phi}} \sim \frac{V^{3/2}}{m_P^3 V'}$$
- CMB: $$\frac{\delta T}{T} \sim \Psi \sim 10^{-5}$$ $$m_P \equiv \frac{1}{\sqrt{8\pi G}}$$

Example: $$V = \frac{1}{2}m^2\phi^2 \quad \Rightarrow \quad \frac{V^{3/2}}{V'} \sim m\phi^2 \Rightarrow m \sim 10^{-5} m_P$$

$$V^{1/4} \sim 10^{16}\text{GeV}$$

Primordial spectra

- Scalar spectrum

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2 \simeq \frac{1}{24\pi^2}\left(\frac{V}{m_{\mathrm{P}}^4\epsilon}\right)_* \simeq 2\times 10^{-9}$$ (COBE)

$$n_s - 1 = \frac{d\ln\mathcal{P}_{\mathcal{R}}}{d\ln k} = -6\epsilon + 2\eta \qquad \epsilon \equiv \frac{m_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1 \qquad \eta \equiv m_P^2\frac{V''}{V} \ll 1$$

Planck : $n_s = 0.965 \pm 0.004 \ (68\,\%\ \mathrm{CL})$

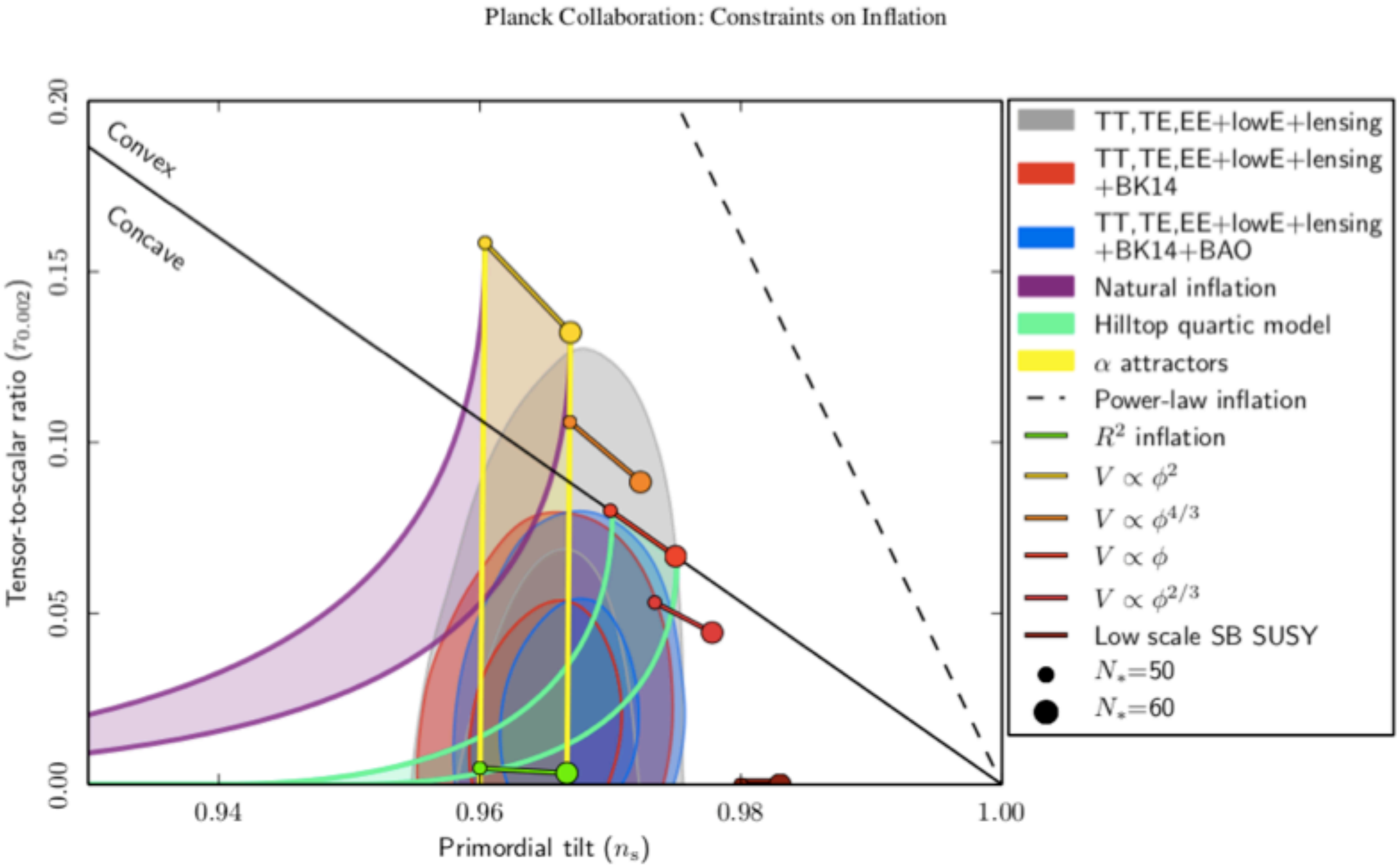
- Gravitational spectrum $\mathcal{P}_T = \frac{2}{3\pi^2}\left(\frac{V_*}{m_{\mathrm{P}}^4}\right)$

- Tensor/scalar ratio: $r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}} = 16\epsilon$

$$V^{1/4} \simeq 10^{16}\mathrm{GeV}\left(\frac{r}{0.01}\right)^{1/4}$$

Planck: $r < 0.07$

Constraints on primordial spectrum



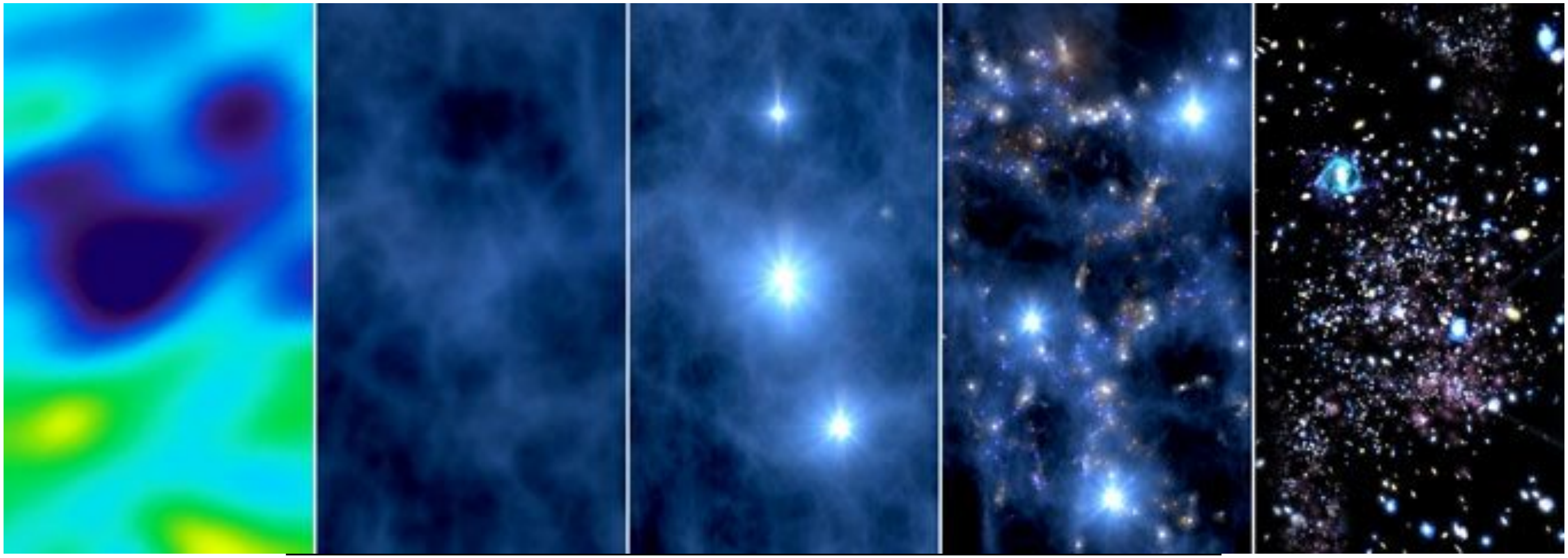
$$n_s = 0.965 \pm 0.004$$

$$r < 0.07$$

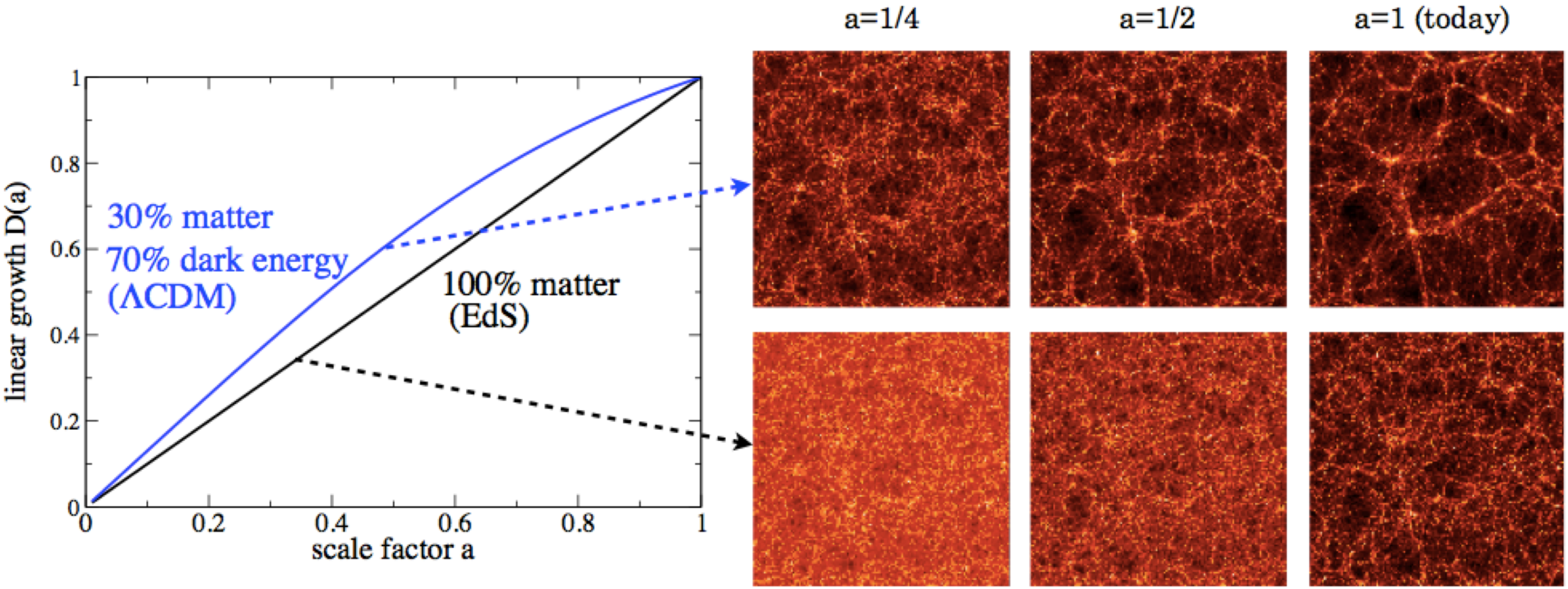
From primordial fluctuations to large-scale structures

- The primordial fluctuations grow because of gravity

- Nonlinear regime: formation of stars, galaxies, clusters…



Structure growth



$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\rho_m\delta_m = 0 \qquad \delta_m(a) = D(a)\,\delta_m(a=1)$$

Beyond the standard model

- **Beyond a cosmological constant ?**
- **Background:** time-evolving dark energy

$$w(a) = \frac{P(a)}{\rho(a)}$$

$$w(a) = w_0 + w_a(1 - a)$$

- **Perturbations:** even if the background evolution is the same as for LCDM, the evolution of perturbations could be different.
- **Consistency** between the expansion history & evolution of perturbations is a strong test for the standard model or for alternative scenarios.

Dark energy or modified gravity

- Cosmological constant ?

$$w = \frac{P}{\rho} \neq -1\,?$$

- Alternative: modified gravity

  Most studied: scalar-tensor theories $\quad g_{\mu\nu}\,, \quad \phi$

$$S_{\rm grav} = \int d^4x\sqrt{-g}\,\{F(X,\phi)R + F_0(X,\phi) + F_1(X,\phi)\Box\phi + \\ + C^{\mu\nu\rho\sigma}(\phi,\nabla\phi)\,\nabla_\mu\nabla_\nu\phi\,\nabla_\rho\nabla_\sigma\phi\,\}$$

$$X \equiv g^{\mu\nu}\nabla_\mu\phi\,\nabla_\nu\phi$$

- Some of these theories predict $\quad c_g \neq c$

  New constraint (GW170817):

$$|c_g - c|/c \lesssim 10^{-15}$$

A new window: gravitational waves

- The perturbations of spacetime

$$ds^2 = \sum_{\mu,\nu} (\eta_{\mu\nu} + h_{\mu\nu})\, dx^\mu dx^\nu \,, \qquad |h_{\mu\nu}| \ll 1$$

can propagate as **gravitational waves**

- The linearization of Einstein's equation $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$

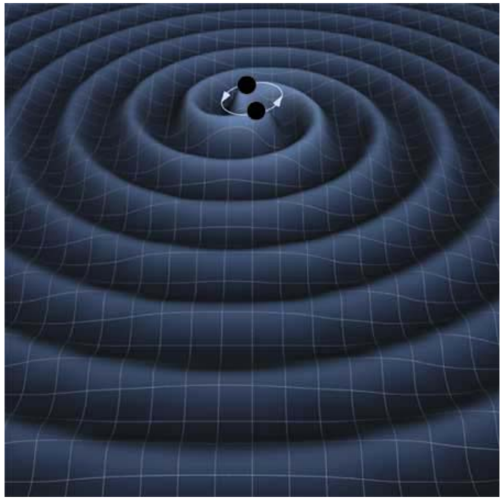
leads to

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \triangle \right) \bar{h}_{\mu\nu} = 16\pi G\, T_{\mu\nu}$$

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2} h\, \eta_{\mu\nu}, \quad h \equiv \eta^{\rho\sigma} h_{\rho\sigma}$$

- Similar to Maxwell's equations

Gravitational waves



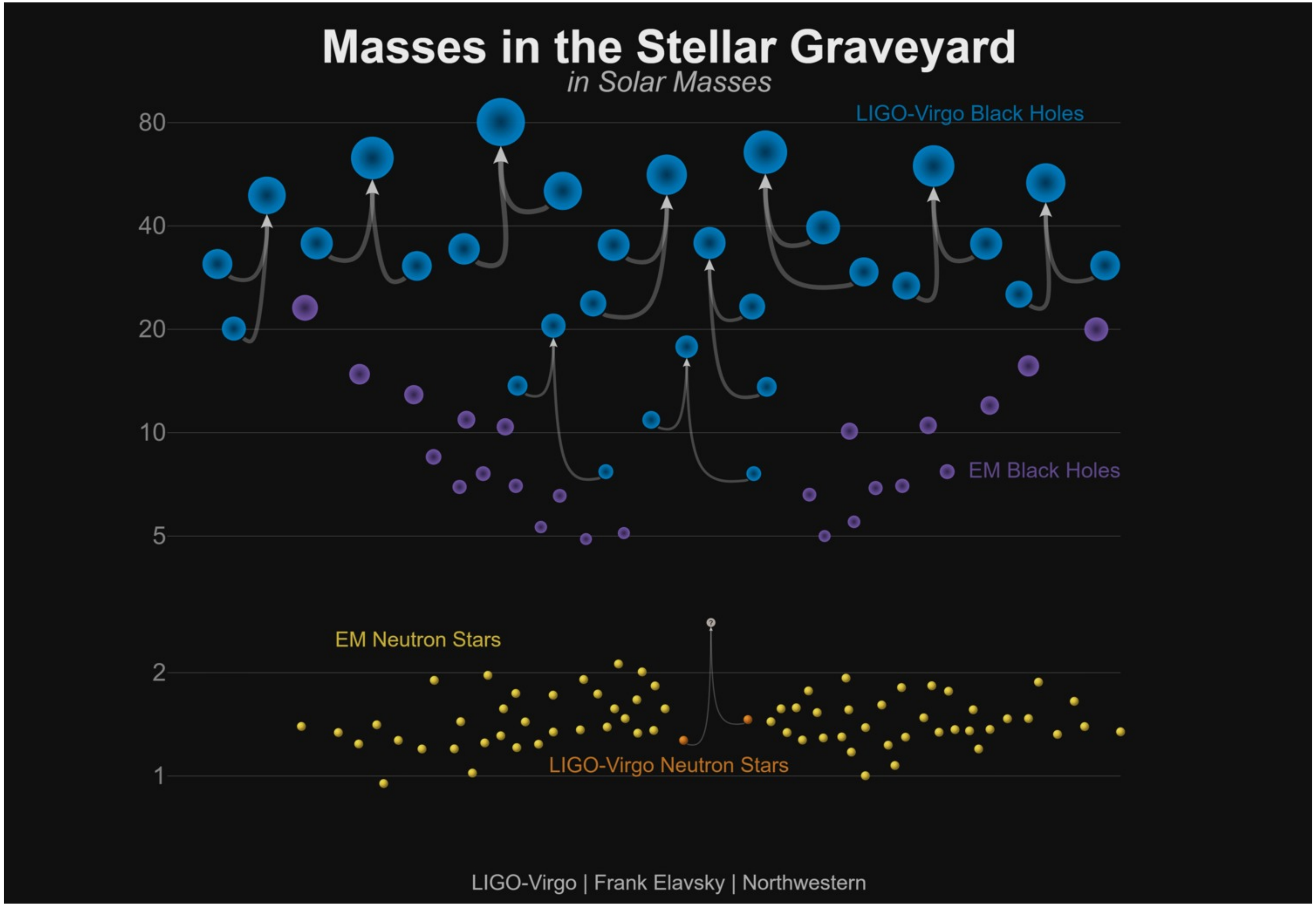
- Binary system

Typically, $D \sim 100\,\mathrm{Mpc} \sim 10^{21}\mathrm{km}\,,$
$R_S \sim 10\,\mathrm{km}\,, d \sim 10\,R_S$

$$h \sim \frac{(GM)^2}{c^4 D d} \sim \frac{R_S^2}{Dd} \sim 10^{-21}$$

- Interferometers (3-4 km arms)
  - LIGO: 2 (USA)
  - Virgo (Italy)
  - Soon KAGRA (Japan)

Detected black holes & neutron stars



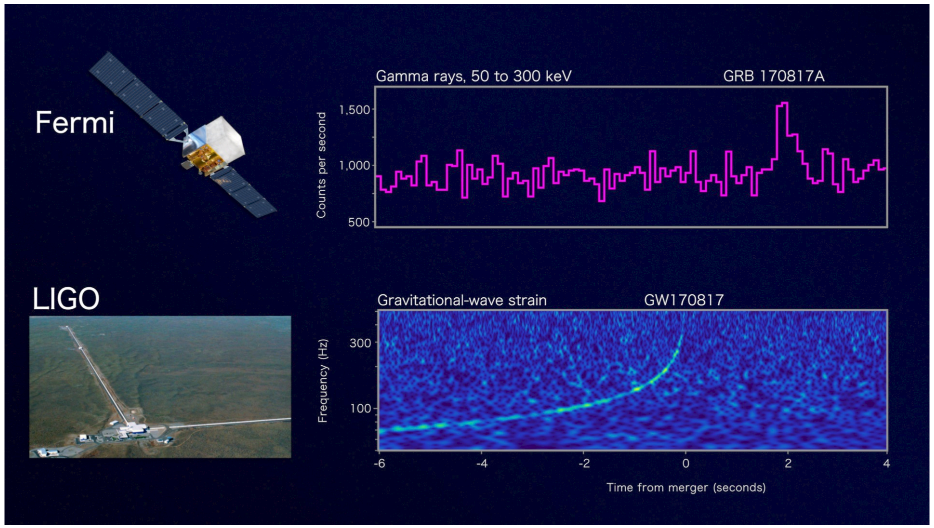
Gravitational waves & cosmology

- The gravitational signal gives a measurement of D_L

$$h \propto \frac{1}{D_L}$$

- An optical counterpart (gamma ray burst) gives the redshift.

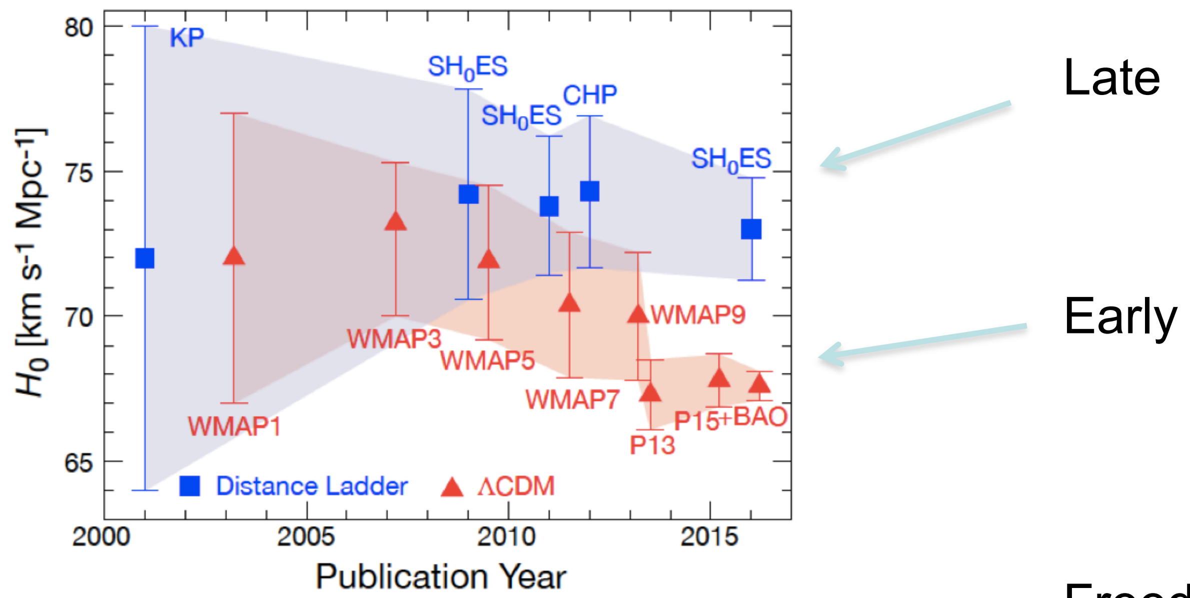
- Event GW170817 & GRB 170817A:



$$H_0 = 70.0^{+12.0}_{-8.0}\,\mathrm{km.s^{-1}Mpc^{-1}}$$

Measurement of H_0

- Original estimate (1929): $H_0 \sim 500\,\mathrm{km.s^{-1}Mpc^{-1}}$
- Today: $H_0 \sim 70\,\mathrm{km.s^{-1}Mpc^{-1}}$



Freedman, 2017

Conclusions

- **Standard cosmological model,** remarkably confirmed by observations (in particular by the Planck satellite)
  - 5 % ordinary matter (baryons)
  - 25 % Cold Dark Matter
  - 70 % cosmological constant

  But there is a tension between early & late measurements of H_0

- **Some (crucial) open questions**:
  - Inflation ?
  - Nature of dark matter ?
  - Cosmological constant, dark energy or modified gravity ?

- The LambdaCDM model and general relativity will be further tested by upcoming experiments